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Resonance tunnelling and breakdown of the quantum Hall effect in strong electric fields

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Abstract. Resonance scattering of electrons on impurities is shown to play a crucial role in non-linear conductivity under quantum Hall effect conditions at zero temperature. The dissipative current as a function of electric field and Landau level filling factor is calculated.

1. Introduction

It is well known that the longitudinal conductivity turns to zero in the quantum Hall effect (QHE) conditions at zero temperature. Nevertheless there exists a longitudinal current depending non-linearity on the electric field E. This current is caused by the interaction of electrons with impurities and phonons. The experiments (Cage *et al* 1983, Ebert *et al* 1983) have shown the longitudinal current to be sharply rising when E exceeds some critical value E_c. Electric field is related to the drift velocity of electrons by $V_d = cE/H$. Cage *et al* (1983) and Ebert *et al* (1983) found that v_d corresponding to E_c is close to the sound velocity s. As a consequence theories (Heinonen *et al* 1984, Streda and von Klitzing 1984, Smrcka 1985) appeared which treated the dissipation as originated by Cherenkov radiation of phonons. The comparison of theoretical results (Heinonen *et al* 1984, Streda and von Klitzing 1984) with the experiment can hardly be done because of the strong inhomogeneity of the current distribution in the samples (Simon *et al* 1986). Moreover, for small samples (Bliek *et al* 1986a, b, d'Iorio *et al* 1987) v_d corresponding to E_c exceeds the velocity of sound by at least an order of magnitude. So, Cherenkov radiation by itself cannot explain the fast increase of dissipative current.

We consider the resonance tunnelling of electrons between Landau levels and show that under certain conditions it plays the leading role in the formation of the non-linear dissipative current. Energy is conserved in tunnelling processes, so the dissipation can be attributed to the electron–phonon interaction. Temperature is supposed to be zero. It means that only emission of phonons by electrons should be taken into account. The thermal conductivity is assumed to be sufficiently large, permitting phonons to go out of the system freely.

This paper is organised in the following way. In § 2 we consider an elementary process of resonance tunnelling through an impurity, and find the conditions that are necessary to neglect the tunnelling through a chain of impurities compared with tunnelling through a single impurity. In § 3 the kinetics of the electron–phonon system is considered. The following processes are included: tunnelling between Landau levels and impurities, jumps of electrons with emission of phonons along one Landau level as well as jumps to

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another Landau level or an impurity. We find the dissipative current as a function of the filling factor and electric field. Section 4 is devoted to fluctuations of the current.

2. Impurity-mediated resonance tunnelling between Landau levels

We consider first an electron moving in the plane (x, y). The magnetic field H is supposed to be perpendicular to this plane. The electric field E is directed along the x axis. In the Landau gauge the vector potential has only one component $A_y(x) = Hx$. So, the y component of the momentum (denoted p) is conserved. In the presence of an electric field the electron wavefunction $\psi_{np}(\boldsymbol{r})$ does not change; however, the energy of the electron state depends on p linearly:

$$\varepsilon_{np} = \hbar\omega_c(n + \tfrac{1}{2}) - eE\,cp/eH \tag{1}$$

where $\varepsilon = eH/mc$ is the cyclotron frequency. We would like to remind readers that $x = cp/eH$ is the mean coordinate of the electron.

Direct transitions of electrons between Landau levels are permitted by the energy conservation law but they are forbidden by the momentum conservation law. Hence electron scattering by impurities or phonons is necessary to permit transitions between Landau levels.

Let us consider tunnelling between Landau levels mediated by impurities. Any impurity has at least one bound state between any Landau levels. A resonance with a bound-state level strongly enhances the tunnelling probability. For a proper description of the resonance tunnelling the exact electron wavefunction in a potential created by the impurity has to be found. We accept the simplest model form of this potential:

$$V(\boldsymbol{r}) = \lambda\delta(\boldsymbol{r}). \tag{2}$$

The electron energy spectrum for such an impurity has been calculated by Prange (1981). Here we find exact wavefunctions for the same problem. More generally, we consider the Hamiltonian $H = H_0 + V(\boldsymbol{r})$ where $V(\boldsymbol{r})$ is defined by equation (2). The Green function for the Hamiltonion H has a form

$$G_\varepsilon(\boldsymbol{r}, \boldsymbol{r}') = G_\varepsilon^0(\boldsymbol{r}, \boldsymbol{r}') + \frac{G_\varepsilon^0(\boldsymbol{r}, 0)\lambda G_\varepsilon^0(0, \boldsymbol{r}')}{1 - \lambda G_\varepsilon^0(0, 0)} \tag{3}$$

where

$$G_\varepsilon(\boldsymbol{r}, \boldsymbol{r}') = \sum_\alpha \frac{\psi_\alpha^0(\boldsymbol{r})\psi_\alpha^{0*}(\boldsymbol{r}')}{\varepsilon - \varepsilon_\alpha + \mathrm{i}0} \tag{4}$$

is the Green function for the Hamiltonian and α labels eigenstates with energy ε_α. By the standard procedure we get the eigenfunctions of the Hamiltonian

$$\psi_\alpha(\boldsymbol{r}, t) = \lim_{t' \to -\infty} \int G_\varepsilon(\boldsymbol{r}, \boldsymbol{r}', t - t')\psi_\alpha^0(\boldsymbol{r}', t)\,\mathrm{d}\boldsymbol{r}'$$
$$= \left(\psi_\alpha^0(\boldsymbol{r}) + \frac{\lambda G_{\varepsilon\alpha}^0(\boldsymbol{r}, 0)}{1 - \lambda G_{\varepsilon\alpha}^0(0, 0)}\psi_\alpha^0(0)\right)\mathrm{e}^{-\mathrm{i}\varepsilon_\alpha t}. \tag{5}$$

The scattering amplitude is equal to the projection of ψ_α onto the unperturbed state ψ_α^0:

$$M_{\alpha\alpha'} = \frac{\lambda\psi_\alpha^0(0)\psi_{\alpha'}^{0*}(0)}{(\varepsilon - \varepsilon_{\alpha'} + \mathrm{i}0)[1 - \lambda G_{\varepsilon\alpha}^0(0, 0)]}. \tag{6}$$

Using equation (6) one finds the transition probability

$$W_{\alpha\alpha'} = \frac{2\pi}{\hbar}\frac{\delta(\varepsilon - \varepsilon_{\alpha'})|\varphi_\alpha|^2|\varphi_{\alpha'}|^2}{|1/\lambda - K(\varepsilon)|^2} \tag{7}$$

where $\varphi_\alpha = \psi_\alpha^0(0)$ and $K(\varepsilon) = G_{\varepsilon\alpha}^0(0, 0)$.

Equation (7) has the typical form of a Breit–Wigner probability for a resonance scattering through a bound state. The energy ε_0 of the bound state is defined by

$$1/\lambda - \operatorname{Re} K(\varepsilon_0) = 0.$$

The values $|\varphi_\alpha|^2$ and $|\varphi_{\alpha'}|^2$ are proportional to probabilities to go into and go out of the bound state. The imaginary part of $K(\varepsilon)$ is proportional to the total probability of decay through all possible channels:

$$\operatorname{Im} K(\varepsilon) = \pi \sum \delta(\varepsilon - \varepsilon_\alpha)|\varphi_\alpha|^2. \tag{8}$$

For our particular problem we should replace α by two parameters: an integer n, which is the number of the Landau levels, and the y component of p. Further we assume $n = 0$ in the initial state and $n = 1$ in the final state. So, the role of α and α' is played by p and p' or corresponding coordinates x and x'. Energy conservation implies the following relationship between $x - x'$ and the electric field E:

$$x' - x \equiv \Delta = \hbar\omega_c/eE. \tag{9}$$

For small electric fields the value of Δ exceeds the magnetic length l_H. Therefore electrons involved in the tunnelling can be considered quasi-classically. This means that a single impurity creates electric current

$$I = e \sum_{x_1 x_2} W_{x_1 x_2}. \tag{10}$$

Here we supposed that the first Landau level is filled almost completely while the second Landau level is almost empty. This is true in small electric fields since the tunnelling current is exponentially small.

The transition probability has a sharp maximum at $\varepsilon_\alpha = \varepsilon_0$. Hence asymptotically at $E \to 0$ we have

$$I = \frac{e\Delta^2}{\hbar} \frac{|\varphi_{0p_0}|^2 |\varphi_{1p_1}|^2}{(\partial K/\partial\varepsilon)|_{\varepsilon=\varepsilon_0} 2 \operatorname{Im} K(\varepsilon_0)} \qquad p_n = \frac{\hbar\Delta}{l_H^2}(\xi + \tfrac{1}{2} - n) \tag{11}$$

where dimensionless parameter $\xi = (\varepsilon_0 - \hbar\omega_c)/\hbar\omega_C$ describes a deviation of the bound-state energy from the middle of the gap between Landau levels. Equation (11) can be rewritten as

$$I = eW_0W_1/(W_0 + W_1) \tag{12}$$

where

$$W_n = \frac{\Delta}{\hbar} \frac{|\varphi_{np_n}|^2}{(\partial K/\partial\varepsilon)|_{\varepsilon=\varepsilon_0}} \tag{13}$$

is the probability of transition for an electron from an impurity to the nth Landau level.

The current (12) takes its maximum when the bound-state energy at zero magnetic field lies just in the middle of the gap between Landau levels. In the vicinity of the maximum it can be rewritten as

$$I = \frac{\sqrt{\pi}}{4(1 + \pi^2/8)} e\omega_c \left(\frac{\Delta}{l_H}\right)^3 \frac{\exp(-\Delta^2/4l_H^2)}{\exp(\xi\Delta^2/l_H^2) + (\Delta^2/2l_H^2)\exp(-\xi\Delta^2/l_H^2)}. \tag{14}$$

In small samples, electrons and holes created by tunnelling processes presumably go out of the system without recombination. Hence, the total current can be presented as a sum of currents (14) generated by single impurities (Pokrovsky *et al* 1988). In large samples, where recombination takes place (see § 3), the dissipative current decreases and equation (14) defines the upper limit of the single impurity contribution to the total dissipative current.

In practice, current is available for a measurement if the ratio Δ/l_H is not too large. For $\Delta/l_h \ll 1$ the current I equals $e\omega_c$ within an order of magnitude, i.e. $I \sim 1\ \mu$A for $H = 10$ T.

The current (14) decays exponentially with increasing $|\xi|$. Assuming the density of states $\rho(\xi + \frac{1}{2})$ to be slowly varying with ξ near the centre of the gap between Landau levels, the mean value of the current produced by one impurity is

$$\bar{I} = \frac{2^{1/2}\pi^{3/2}}{\pi^2 + 8}\rho(\tfrac{1}{2})e\omega_c \exp\left(-\frac{\Delta^2}{4l_H^2}\right). \tag{15}$$

Tavger and Erukhimov (1966) were the first to consider the problem of tunnelling between Landau levels. They studied non-resonance tunnelling and got much smaller probability proportional to $\exp(-\frac{1}{2}\Delta^2/l_H^2)$. Chaplik and Entin (1974) and Lifshiz and Kirpichenkov (1979) considered resonance tunnelling in the absence of a magnetic field.

To get (15) we assumed that the tunnelling through a fixed impurity is not influenced by other impurities. That will be the case for small enough concentration of impurities. To estimate a proper range of concentrations we calculate the probability of tunnelling mediated by a pair of impurities having strengths λ_1 and λ_2 and placed at points $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. The Green function of an electron in the field created by these impurities is

$$G_\varepsilon(\boldsymbol{r}, \boldsymbol{r}') = G_\varepsilon^0(\boldsymbol{r}, \boldsymbol{r}') + \sum_{i,j} G_\varepsilon^0(\boldsymbol{r}, \boldsymbol{r}_i)M_{ij}^{-1}G_\varepsilon^0(\boldsymbol{r}_j, \boldsymbol{r}') \tag{16}$$

where

$$\mathbf{M} = \left\| \begin{matrix} 1/\lambda_1 - G_\varepsilon^0(\boldsymbol{r}_1\boldsymbol{r}_1) & -G_\varepsilon^0(\boldsymbol{r}_1\boldsymbol{r}_2) \\ -G_\varepsilon^0(\boldsymbol{r}_2\boldsymbol{r}_1) & 1/\lambda_2 - G_\varepsilon^0(\boldsymbol{r}_2\boldsymbol{r}_2) \end{matrix} \right\|. \tag{17}$$

The probability for an electron with initial momentum p to get from the first Landau level to the second one acquiring the final momentum p' is

$$W_{pp'} = \frac{2\pi}{\hbar}\delta(\varepsilon_{0p} - \varepsilon_{1p'})\left|\sum_{ij}\psi_{0p}^{0*}(\boldsymbol{r}_i)M_{ij}^{-1}\psi_{1p'}^0(\boldsymbol{r}_j)\right|^2. \tag{18}$$

The probability (18) reaches its maximum for pairs of impurities having close values of y coordinates. The bound-state energies associated with such pairs placed in a given electric field should also be close to each other. The tunnelling current created by such a pair of impurities is given by a sum of $W_{pp'}$ over all the initial and final states:

$$I = \int \frac{\mathrm{d}p\,\mathrm{d}p'}{(2\pi\hbar)^2}\frac{2\pi}{\hbar}\frac{\delta(\varepsilon_{0p} - \varepsilon_{1p'})|\psi_{0p}^{0*}(\boldsymbol{r}_1)G_{\varepsilon p}^0(\boldsymbol{r}_1\boldsymbol{r}_2)\psi_{1p'}^0(\boldsymbol{r}_2)|^2}{|\det \mathbf{M}|^2}. \tag{19}$$

The denominator on the RHS of equation (19) becomes very small at the energy close to bound-state energy

$$1/\lambda_1 = \mathrm{Re}\, G_{\varepsilon_p}^0(\boldsymbol{r}_1\boldsymbol{r}_1) \qquad 1/\lambda_2 = \mathrm{Re}\, G_{\varepsilon_p}^0(\boldsymbol{r}_2\boldsymbol{r}_2). \tag{20}$$

The Green function $G_{\varepsilon_p}^0(\boldsymbol{rr})$ depends on $\boldsymbol{r}$ since a system is influenced by electric field. The current (19) averaged over the bound-state energies takes the form:

$$I = \frac{\pi}{\Delta\hbar^3}\int \mathrm{d}p\,\mathrm{d}p' \frac{\delta(\varepsilon_{0p} - \varepsilon_{1p'})\rho_1(\varepsilon_p)\rho_2(\varepsilon_{p'})}{[\partial G_\varepsilon^0(\boldsymbol{r}_1\boldsymbol{r}_1)/\partial\varepsilon]|_{\varepsilon=\varepsilon_p}[\partial G_\varepsilon^0(\boldsymbol{r}_2\boldsymbol{r}_2)/\partial\varepsilon]|_{\varepsilon=\varepsilon_p}} \times \frac{|\psi_{0p}^0(\boldsymbol{r}_1)|\,|G_{\varepsilon_p}^0(\boldsymbol{r}_1\boldsymbol{r}_2)|^2\,|\psi_{1p'}^0(\boldsymbol{r}_2)|}{[(\Delta/2)^2|\psi_{0p}^0(\boldsymbol{r}_1)\psi_{1p'}^0(\boldsymbol{r}_2)|^2 + |G_{\varepsilon_p}^0(\boldsymbol{r}_1\boldsymbol{r}_2)|^2]^{1/2}} \tag{21}$$

where $\rho_i(\varepsilon) = \rho(\varepsilon + x_i/\Delta - \frac{1}{2})$. The expression (21) has a maximum when the distance between impurities is equal to $\Delta/2$. Averaging over the positions of impurities we get

$$I \propto c\rho(\tfrac{1}{4})\rho(\tfrac{3}{4}) \exp[-\tfrac{1}{8}(\Delta/l_H)^2] \tag{22}$$

where c is the concentration of impurities. Comparing this result with the current (15) associated with single impurities one finds that the latter prevails provided the following inequality is satisfied:

$$\rho(\tfrac{1}{4})\rho(\tfrac{3}{4})cl_H^2 \ll \rho(\tfrac{1}{2}) \exp[-\tfrac{1}{8}(\Delta/l_H)^2]. \tag{23}$$

For a fixed concentration of impurities the condition (23) can be broken when decreasing the electric field. Then the tunnelling will proceed through chains of impurities. A similar situation has been considered by Shklovskii (1982).

3. Dissipative current as a function of Landau level filling

The interaction of an electron with impurities cannot change its energy. To get the dissipative current the electron–phonon processes should be included. As before we consider the case of zero temperature with no living phonons. Hence, scattering and absorption of phonons by electrons does not take place, only the emission of phonons is possible.

For $v_d > s$ the conservation laws permit electrons to emit phonons and to move along Landau levels. On the other hand, if $v_d < s$ in a pure system it is impossible. However, in the presence of impurities breaking the momentum conservation, electrons can transit between different states of the same Landau level even for $v_d < s$.

Owing to the tunnelling electrons appear at the second Landau level and holes appear at the first. They can recombine with the emission of phonons. Recombination is probable for an electron and a hole having the same coordinates with uncertainty l_H and the corresponding probability does not contain an exponentially small factor. Recombination opens an additional channel of the decay of a bound state on an impurity. The width of the resonance level increases, in turn decreasing the resonance tunnelling probability. This is still correct even when the number of electrons on the second Landau level and holes on the first one is exponentially small.

Assuming the electron–phonon interaction to be weak we neglect processes with more than one phonon emitted and phonon-assisted tunnelling. We also neglect transitions of electrons from one localised state to another. As has been explained above, it holds if the impurity concentration is small enough, and/or the electric field is sufficiently high. Under these assumptions the following system of kinetic equations for the electron density $N_e(\boldsymbol{r})$ and the hole density $N_h(\boldsymbol{r})$ can be written:

$$\begin{aligned}\partial N_e/\partial t = {} & \sum_{r'} P_e(\boldsymbol{r'r})N_e(\boldsymbol{r'}) - N_e(\boldsymbol{r})\sum_{r'} P_e(\boldsymbol{rr'}) - v_d\, \partial N_e/\partial y \\ & + \sum_i n_i W_e(i,\boldsymbol{r}) - N_e(\boldsymbol{r})\sum_i (1-n_i)Q_e(i,\boldsymbol{r}) \\ & - N_e(\boldsymbol{r})\sum_{r'} N_h(\boldsymbol{r'})Q_h(\boldsymbol{rr'}) \end{aligned} \tag{24}$$

$$\begin{aligned}\partial N_h/\partial t = {} & \sum_{r'} P_h(\boldsymbol{r'r})N_h(\boldsymbol{r'}) - N_h(\boldsymbol{r})\sum_{r'} P_h(\boldsymbol{rr'}) - v_d\, \partial N_h/\partial y \\ & + \sum_i (1-n_i)W_h(i,\boldsymbol{r}) - N_h(\boldsymbol{r})\sum_i n_i Q_h(i,\boldsymbol{r}) \end{aligned}$$

$$- N_h(r) \sum_{r'} N_e(r') Q_e(r'r). \qquad (25)$$

Here $P_e(rr')$ is the probability of electron scattering on the second Landau level. The function $W_e(i, r)$ is the probability of tunnelling from ith impurity to the second Landau level. $Q_e(i, r)$ is the probability for an electron to fall from the second Landau level to ith impurity, emitting a phonon. Analogous values with index h are relevant to holes on the first Landau level. The value $Q(rr')$ is the probability of direct electron–hole recombination. The variable n_i is the electron occupation number for ith impurity. It is governed by the following kinetic equation:

$$\partial n_i/\partial t = - n_i \sum_r W_e(i, r) + (1 - n_i) \sum_r W_h(i, r) - n_i \sum_r N_h(r) Q_h(i, r) + (1 - n_i) \sum_r N_e(r) Q_e(i, r). \qquad (26)$$

We assume impurities to be distributed homogeneously on average. Then in a large sample stationary densities of electrons and holes do not depend on coordinates. Let $n(\varepsilon)$ be the electron occupation number in a bound state with energy ε. For a stationary homogeneous state the kinetic equations (24)–(26) can be simplified to

$$c \int n(\varepsilon)\rho(\varepsilon)W_e(\varepsilon)\, d\varepsilon - cN_e \int [1 - n(\varepsilon)]\rho(\varepsilon)Q_e(\varepsilon)\, d\varepsilon - N_e N_h Q = 0 \qquad (27)$$

and

$$c \int [1 - n(\varepsilon)]\rho(\varepsilon)W_h(\varepsilon)\, d\varepsilon - cN_h \int n(\varepsilon)\rho(\varepsilon)Q_h(\varepsilon)\, d\varepsilon - N_e N_h Q = 0 \qquad (28)$$

with

$$n(\varepsilon) = \frac{W_h(\varepsilon) + N_e Q_e(\varepsilon)}{W_e(\varepsilon) + W_h(\varepsilon) + N_e Q_e(\varepsilon) + N_h Q_h(\varepsilon)} \qquad (29)$$

where the tunnelling probabilities $W_e(\varepsilon)$ and $W_h(\varepsilon)$ are defined by equation (13) with $n = 1$ and 0 respectively. The value $Q_e(\varepsilon)$ is the total probability of the electron transition from the second Landau level to a bound state on an impurity accompanied by the emission of a phonon. Q_h is the analogous probability for a hole. For given transition probabilities the calculation of the dissipative current is straightforward:

$$I = \eta_e N_e + \eta_h N_h + c\Delta \int \rho(\varepsilon)\{n(\varepsilon)(1 - \varepsilon)W_e(\varepsilon) + [1 - n(\varepsilon)]\varepsilon W_h(\varepsilon)\}\, d\varepsilon \qquad (30)$$

where

$$\eta_e = - \sum_r x P_e(r, 0) \qquad \text{and} \qquad \eta_h = - \sum_r x P_h(0, r).$$

Only two of the three equations (27)–(29) are independent. So, we need additional information to solve this system. In the experiment the filling factor ν or in the other words the total electron density N is usually fixed:

$$N = N_e - N_h + c \int \rho(\varepsilon) n(\varepsilon)\, d\varepsilon. \qquad (31)$$

Equations (28), (29) and (31) define the parametric dependence of the dissipative current on the total electron density N, the values N_e and N_h serving as parameters.

The breakdown of the QHE has been considered previously by Pokrovsky *et al* (1988). The distributions $N_e(x)$ and $N_h(x)$ found in Pokrovsky *et al* (1988) depend on x explicitly. It is true in sufficiently small samples with size L_x along the x axis much less than the electric charge screening length.

Let the number of electrons N be less than the number of bound states on impurities. One should expect that in small electric fields the function $n(\varepsilon)$ is almost the same as for $E = 0$, i.e. the bound states with $\varepsilon < \varepsilon_0(N)$ are filled. If ε is not very close to one of the Landau levels then N_e and N_h are exponentially small. This agrees with equations (27)–(29). According to equation (29) the position of the step in the distribution $n(\varepsilon)$ is smeared out over the interval l_H^2/Δ^2 near ε_0.

An explicit solution of the system (28), (29) and (31) can be found for two limiting cases: when the system is near the centre and near the edge of the Hall plateau.

The centre of the Hall plateau corresponds to $\varepsilon_0 = \frac{1}{2}$. In equations (27)–(29) only the tunnelling probabilities change rapidly with energy ε. Near $\varepsilon = \frac{1}{2}$ they can be written in the form

$$W_e = \gamma(\Delta^2/2l_H^2)\exp[(-\Delta^2/4l_H^2)(1 - 4\xi)] \tag{32}$$

$$W_h = \gamma(\Delta^2/2l_H^2)\exp[(-\Delta^2/4l_H^2)(1 + 4\xi)] \tag{33}$$

where $\varepsilon = \frac{1}{2} + \xi$ as before. The constant γ can be found from equation (13). We replace slowly varying functions $\rho(\varepsilon)$, $Q_e(\varepsilon)$ and $Q_h(\varepsilon)$ by their values at $\varepsilon = \frac{1}{2}$.

We are interested in a regime when extra electrons, added to the system by change of N, stick presumably to impurity levels. It is correct if the impurity concentration satisfies the inequality

$$\exp(-\Delta^2/4l_H) \ll cl_H^2 \ll \exp(-\Delta^2/2l_H^2). \tag{34}$$

Introducing the parameters α and β instead of N_e and N_h by equations

$$N_e Q_e(\tfrac{1}{2}) = \alpha\gamma(\Delta/\sqrt{2}\, l_H)\exp(-\Delta^2/4l_H^2)$$

and

$$N_h Q_h(\tfrac{1}{2}) = \beta\gamma(\Delta/\sqrt{2}\, l_H)\exp(-\Delta^2/4l_H^2) \tag{35}$$

the system of equations (27)–(29) can be reduced to one equation:

$$g(1 - \alpha\beta)\int_0^\infty \frac{\mathrm{d}y}{y^2 + y(\alpha + \beta) + 1} = \alpha\beta \tag{36}$$

where

$$g = c\rho(\tfrac{1}{2})\frac{Q_e(\frac{1}{2})Q_h(\frac{1}{2})}{Q}\frac{\sqrt{2}}{\gamma}\frac{\exp(\Delta^2/4l_H^2)}{(\Delta/l_H)^3}. \tag{37}$$

We suppose that $g \gg 1$. Then near the centre of the plateau it follows from (36) that $\alpha\beta = 1$. In this case equation (31) can be reduced to

$$N = c\int_0^{1/2} \rho(\varepsilon)\,\mathrm{d}\varepsilon + c\rho(\tfrac{1}{2})(l_H^2/\Delta^2)\ln(\sqrt{2}\alpha l_H/\Delta). \tag{38}$$

The first term on the RHS of (38) corresponds to the value of N in the centre of the Hall plateau at zero E. The dissipative current in the same approximation is

$$I = \eta_e N_e + \eta_h N_h = \gamma\frac{\Delta}{\sqrt{2}l_H}\exp\left(-\frac{\Delta^2}{4l_H^2}\right)\left(\frac{\eta_e}{Q_e(\frac{1}{2})}\alpha + \frac{\eta_h}{Q_h(\frac{1}{2})}\frac{1}{\alpha}\right). \tag{39}$$

The dissipative current as a function of the filling factor takes its minimum

$$I_{\min} = \gamma\frac{\sqrt{2}}{l_H}\exp\left(-\frac{\Delta^2}{4l_H^2}\right)\left(\frac{\eta_e\eta_h}{Q_e(\frac{1}{2})Q_h(\frac{1}{2})}\right)^{1/2} \tag{40}$$

at the value of N which depends on electric field

$$N_{\min} = c\int_0^{1/2} \rho(\varepsilon)\,\mathrm{d}\varepsilon + c\rho(\tfrac{1}{2})\frac{l_H^2}{2\Delta^2}\ln\left(\frac{2l_H^2}{\Delta^2}\frac{\eta_h Q_h}{\eta_e Q_e}\right). \tag{41}$$

Consider now the filling of impurity levels near the edge of the Hall plateau. Provided the condition

$$cl_H^2 \gg \exp(-\Delta^2/4l_H^2) \tag{42}$$

is satisfied, the number of holes can be neglected, i.e. N_h and W_h can be put equal to zero in equation (29). Then the mean numbers of electrons on impurity levels are

$$n(\varepsilon) = N_e Q_e(\varepsilon)/[W_e(\varepsilon) + N_e Q_e(\varepsilon)]. \tag{43}$$

The tunnelling probability $W_e(\varepsilon) \propto \exp[-\Delta^2(1-\varepsilon)^2/l_H^2]$ depends exponentially on ε. For $W_e(\varepsilon) < N_e Q_e(\varepsilon)$ the impurity levels are practically completely filled while for $W_e > N_e Q_e(\varepsilon)$ they are empty. Define the energy ε_0 by

$$W_e(\varepsilon_0) = N_e Q_e(\varepsilon_0). \tag{44}$$

The width of an interval in which $n(\varepsilon)$ drops from one to zero is of the order of

$$l_H^2/[\Delta^2(1-\varepsilon_0)]. \tag{45}$$

The total electron density is

$$N = c\int_0^{\varepsilon_0} \rho(\varepsilon)\,\mathrm{d}\varepsilon. \tag{46}$$

Here we have neglected the filling of the second Landau level. As follows from (44) this can be done if

$$\rho(\varepsilon_0)cl_H^2 \gg \exp[-\Delta^2(1-\varepsilon_0)^2/l_H^2]. \tag{47}$$

Near the plateau edge only the transitions of electrons between Landau level and impurity levels have been taken into account. We neglected the transitions from one impurity to another or multiple scattering by impurities. This requires the concentration of impurities to be sufficiently small, i.e.

$$\rho(\varepsilon_0)cl_H^2 \ll \exp[-\Delta^2(1-\varepsilon_0)^2/2l_H^2]. \tag{48}$$

The constraints (47) and (48) can be satisfied for larger $\rho(\varepsilon_0)$ than (34). The reason is that near the plateau edge the tunnelling distance is much less than near the centre.

The dissipative current is defined by the first term of equation (30). It is equal to

$$I = [\eta_e/Q_e(\varepsilon_0)]W_e(\varepsilon_0). \tag{49}$$

Energy ε increases with N according to (46) until inequality (47) is true. When increasing N further, delocalised states at the upper Landau level will be occupied, ε_0 will remain constant. This means that the dissipative current rises sharply after filling of impurity levels with $\varepsilon = \varepsilon_0$ where ε_0 is defined by

$$1-\varepsilon_0 \sim (l_H/\Delta)[\ln(1/cl_H^2)]^{1/2}. \tag{50}$$

In this sense the width of the plateau decreases linearly with increase of electric field. The linear dependence of the plateau width on electric field has general character and is caused exclusively by the sharp dependence of the tunnelling probability on the tunnelling length.

4. Fluctuations

Very small samples with a size of the order of 1 μm have been used in experiments (Bliek *et al* 1986a, b, d'Iorio *et al* 1987). This size is still much larger than the screening length (about 400 Å according to an estimate by Kane *et al* (1988)). Therefore the electron density under the experimental conditions fluctuates weakly. The total number of electrons in a sample was about 10^4. The number of impurities, especially strong impurities, is much less than the number of electrons. Noticeable fluctuations of $I_{\rm diss}(N)$ can be expected. They are described by a formula

$$\mathrm{d}\ln I/\mathrm{d}N = 2(\Delta^2/l_H^2)(1-\varepsilon_0)/c\rho(\varepsilon_0) \tag{51}$$

following from (46) and (49).

Because of the finite width of the step of $n(\varepsilon)$ near ε_0 the value $\rho(\varepsilon_0)$, entering (51), should be averaged over the energy interval:

$$\delta\varepsilon \simeq l_H^2/[\Delta^2(1-\varepsilon_0)].$$

Equation (51) shows that fluctuations of $\rho(\varepsilon)$ result in fluctuations of $I(N)$. Let $\rho(\varepsilon)$ have a peak at energy ε_1. Then, in some interval of variation of N, additional electrons fill impurity levels having energy ε_1. So the current, defined by equation (49), remains constant. There appears a step on the curve $I(N)$. To define the position of such a step as a function of E we should use the more exact relationship between N and ε_0 instead of (46):

$$N = c\int_0^{\varepsilon_0} \rho(\varepsilon)\,\mathrm{d}\varepsilon + c\rho(\varepsilon_0)\frac{\pi^2/24}{(1-\varepsilon_0)^3(\Delta/l_H)^4}. \tag{52}$$

So the location of the step is defined by

$$\delta N \propto E^4. \tag{53}$$

This agrees qualitatively with the motion of steps found by Bliek *et al* (1986b).

The experimental situation is ambiguous. In the first experiments Bliek *et al* (1986b) have claimed the observation of many steps on the $I(H)$ curve. However, this was not confirmed in following publications (Bliek *et al* 1986a). d'Iorio *et al* (1987) stated that they really have observed steps on the same curve, but these steps are not so well pronounced. Two important features of fluctuations observed by Bliek *et al* (1986b) were their reproducibility for a given sample and different curves for different samples. This feature is typical for mesoscopic fluctuations, when only few impurities contribute to the current.

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